

FACOLTÀ DI SCIENZE MM.FF.NN.
Corso di laurea in Scienze Biologiche

Prova scritta di Matematica
del 18-02-2010

Cognome..... Nome..... N° Matricola.....

1. Studiare la seguente funzione:

$$f(x) = (2+x) e^{-\frac{2}{5}x^2} + 2.$$

calcolati
con il segno e m H

2. Calcolare il seguente integrale indefinito:

$$\int e^{3x} \sin(3x) dx. \text{ FATTO}$$

- OK 3. Calcolare il seguente limite:

$$\lim_{x \rightarrow 0} \left[\frac{3 - 3 \cos^2 x}{2x^2} + (x - 1) \right]. \text{ Svolto}$$

4. Dimostrare e commentare il seguente teorema:

Ogni due antiderivate di una funzione, in un fissato intervallo, differiscono per una costante.

- OK 5. Calcolare la derivata della seguente funzione:

$$f(x) = 3x \cos(\lg x) + 4x \sin(\log x) - 2x.$$

$$f(x) = (2+x) e^{-\frac{2}{5}x^2} + 2 \quad (D)]-\infty; +\infty[$$

(S) $(2+x) e^{-\frac{2}{5}x^2} + 2 > 0$

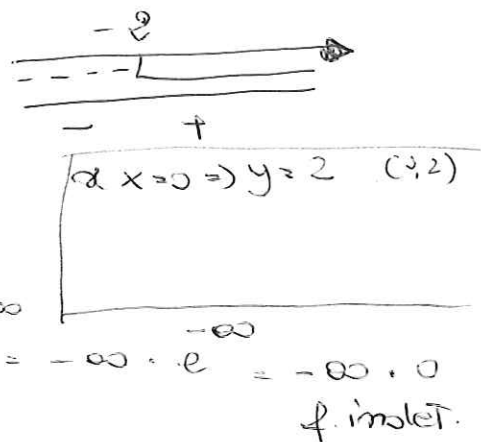
osservo: lo studio del segno presenta difficoltà algebriche.
Decido di studiare la funzione $g(x) = (2+x) e^{-\frac{2}{5}x^2}$
e di traslare successivamente tutti i risultati
verso l'alto di 2 unità.

Infatti $f(x) = g(x) + 2$.

$$g(x) = (2+x) e^{-\frac{2}{5}x^2} \quad D.]-\infty; +\infty[$$

(S) $(2+x) e^{-\frac{2}{5}x^2} > 0$ $\begin{cases} 2+x > 0 \Rightarrow x > -2 \\ e^{-\frac{2}{5}x^2} > 0 \text{ sempre} \end{cases}$

(la f. exp.
è sempre positiva)



(L) $\lim_{x \rightarrow -\infty} (2+x) e^{-\frac{2}{5}x^2} = -\infty \cdot e^{-\frac{2}{5}(-\infty)^2} = -\infty \cdot e^{-\frac{2}{5} \cdot \infty} = -\infty \cdot e^{-\infty} = -\infty \cdot 0$
f. indet.

osservo che $e^{-\frac{2}{5}x^2} = \frac{1}{e^{\frac{2}{5}x^2}}$

Allora $\lim_{x \rightarrow -\infty} (x+2) e^{-\frac{2}{5}x^2} = \lim_{x \rightarrow -\infty} \frac{x+2}{e^{\frac{2}{5}x^2}} = \frac{-\infty}{+\infty} = \frac{-\infty}{+\infty} = 0^-$

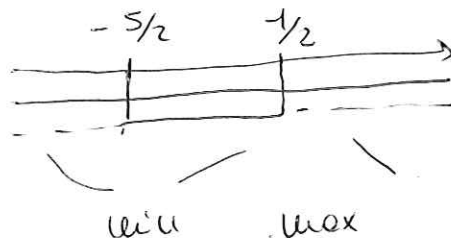
$= \lim_{x \rightarrow -\infty} \frac{(x+2)'}{(e^{\frac{2}{5}x^2})'} = \lim_{x \rightarrow -\infty} \frac{1}{e^{\frac{2}{5}x^2} \cdot \frac{2}{5} \cdot 2x} = \lim_{x \rightarrow -\infty} \frac{1}{\frac{4}{5}x e^{\frac{2}{5}x^2}} = \frac{1}{-\infty \cdot e^{+\infty}} = \frac{1}{-\infty \cdot \infty} = \frac{1}{-\infty} = 0^-$
A.O.

$\lim_{x \rightarrow +\infty} (2+x) e^{-\frac{2}{5}x^2} = \lim_{x \rightarrow +\infty} \frac{2+x}{e^{\frac{2}{5}x^2}} = \frac{\infty}{\infty} = \frac{1}{\frac{4}{5}x e^{\frac{2}{5}x^2}} = \frac{1}{\infty} = 0^+$

D.P.R.T.A $y' = e^{-\frac{2}{5}x^2} + (2+x) e^{-\frac{2}{5}x^2} \cdot \left(-\frac{4}{5}x\right) = e^{-\frac{2}{5}x^2} \left(1 + (2+x)\left(-\frac{4}{5}x\right)\right) =$
 $= e^{-\frac{2}{5}x^2} \left(1 - \frac{4}{5}x \cdot 2 - \frac{4}{5}x^2\right) = e^{-\frac{2}{5}x^2} \left(-\frac{4}{5}x^2 - \frac{8}{5}x + 1\right) > 0$ $\begin{cases} e^{-\frac{2}{5}x^2} > 0 \quad \forall x \\ -\frac{4}{5}x^2 - \frac{8}{5}x + 1 > 0 \end{cases}$

$$-\frac{4}{5}x^2 - \frac{8}{5}x + 1 = 0 \Rightarrow -4x^2 - 8x + 5 = 0$$

$$x_{1/2} = \frac{8 \pm \sqrt{64 + 80}}{-8} = \frac{8 \pm \sqrt{12}}{-8} = \begin{cases} \frac{8+12}{-8} = -\frac{20}{8} = -\frac{5}{2} \\ \frac{8-12}{-8} = \frac{4}{8} = \frac{1}{2} \end{cases}$$



$$x = -\frac{5}{2} \Rightarrow y = (2 - \frac{5}{2}) e^{-\frac{2}{5} \cdot (-\frac{5}{2})^2} = \frac{4-5}{2} e^{-\frac{2}{5} \cdot \frac{25}{4}} = -\frac{1}{2} \cdot e^{-\frac{5}{2}} \approx -0,04$$

$m(-\frac{5}{2}; -\frac{1}{2}e^{-\frac{5}{2}})$ punto di minimo

$$x = \frac{1}{2} \Rightarrow y = (2 + \frac{1}{2}) e^{-\frac{2}{5}(\frac{1}{2})^2} = \frac{4+1}{2} e^{-\frac{2}{5} \cdot \frac{1}{4}} = \frac{5}{2} e^{-\frac{1}{10}} \approx 2,26$$

$M(\frac{1}{2}; \frac{5}{2}e^{-\frac{1}{10}})$ punto di massimo

D. SECONDA

$$\begin{aligned} y'' &= -\frac{4}{5}x e^{-\frac{2}{5}x^2} \left(-\frac{4}{5}x^2 - \frac{8}{5}x + 1\right) + e^{-\frac{2}{5}x^2} \left(-\frac{8}{5}x - \frac{8}{5}\right) = \\ &= e^{-\frac{2}{5}x^2} \left(-\frac{4}{5}x \left(-\frac{4}{5}x^2 - \frac{8}{5}x + 1\right) - \frac{8}{5}x - \frac{8}{5}\right) = e^{-\frac{2}{5}x^2} \left(\frac{16}{25}x^3 + \frac{32}{25}x^2 - \frac{4}{5}x - \frac{8}{5}x - \frac{8}{5}\right) = \\ &= e^{-\frac{2}{5}x^2} \left(\frac{16}{25}x^3 + \frac{32}{25}x^2 - \frac{12}{5}x - \frac{8}{5}\right) \geq 0 \end{aligned}$$

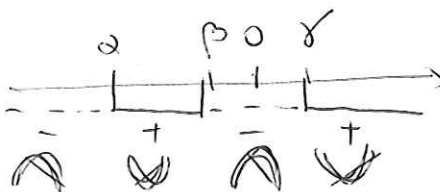
$$\begin{cases} e^{-\frac{2}{5}x^2} \geq 0 \quad \forall x \\ \frac{16}{25}x^3 + \frac{32}{25}x^2 - \frac{12}{5}x - \frac{8}{5} \geq 0 \end{cases}$$

$$\frac{16x^3 + 32x^2 - 40x - 60}{25} \geq 0 \quad \begin{cases} 25 > 0 \quad \forall x \\ 16x^3 + 32x^2 - 40x - 60 \geq 0 \end{cases}$$

divido per 4 tutti i membri

$$\Rightarrow 4x^3 + 8x^2 - 10x - 15 \geq 0$$

$S(x)$



$$S(x) y = 4x^3 + 8x^2 - 15x - 10$$

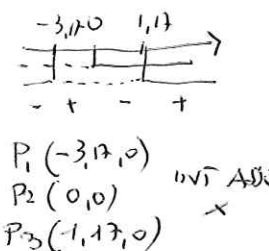
Essendo $4x^3 + 8x^2 - 15x - 10$ difficile da studiare allora ando la flet

$$h(x) = 4x^3 + 8x^2 - 15x \quad \text{e poi la traslo di } -10 \text{ sulle } y$$

$$h(x) = 4x^3 + 8x^2 - 15x \quad (D):]-\infty; +\infty[$$

$$(S) \quad 4x^3 + 8x^2 - 15x \geq 0 \quad x(4x^2 + 8x - 15) \geq 0 \quad \left\{ \begin{array}{l} x \geq 0 \\ 4x^2 + 8x - 15 \geq 0 \end{array} \right.$$

$$4x^2 + 8x - 15 = 0 \quad x_{1/2} = \frac{-8 \pm \sqrt{64 + 240}}{8} = \frac{-8 \pm \sqrt{304}}{8} = \frac{-8 \pm 17,6}{8} = \begin{cases} 1,17 \\ -3,17 \end{cases}$$



$$(L) \quad \lim_{x \rightarrow +\infty} = +\infty \quad \lim_{x \rightarrow -\infty} = -\infty$$

$$(D) \quad y' = 12x^2 + 16x - 15 \geq 0 \quad x_{1/2} = \frac{-16 \pm \sqrt{256 + 720}}{24} = \frac{-16 \pm \sqrt{976}}{24} = \frac{-16 \pm 31,2}{24} = \begin{cases} 0,63 \\ -1,96 \end{cases}$$

$$x = 0,63 \Rightarrow h(0,63) = 4 \cdot (0,63)^3 + 8 \cdot (0,63)^2 - 15 \cdot (0,63) = 1,00 + 3,17 - 9,45 = -5,28$$

$$x = -1,96 \Rightarrow h(-1,96) = 4 \cdot (-1,96)^3 + 8 \cdot (-1,96)^2 - 15 \cdot (-1,96) = -30,11 + 30,73 + 29,4 = 30,02$$

$$m(0,63; -5,28) \text{ minimo} \quad M(-1,96; 30,02) \text{ massimo}$$

$$(D'') \quad y'' = 24x + 16 \geq 0 \quad 24x \geq -16 \Rightarrow x \geq \frac{-16}{24} = -\frac{2}{3} = -0,67$$

$$x = -\frac{2}{3} \Rightarrow y = 4\left(-\frac{2}{3}\right)^3 + 8\left(-\frac{2}{3}\right)^2 - 15\left(-\frac{2}{3}\right) - 10 =$$

$$= 4\left(-\frac{8}{27}\right) - \frac{32}{72} + 10 - 10 = -\frac{32}{27} - \frac{32}{72} = -1,18 - 0,44 = -1,62$$

$$F\left(-\frac{2}{3}; 1,62\right) \text{ flesso}$$

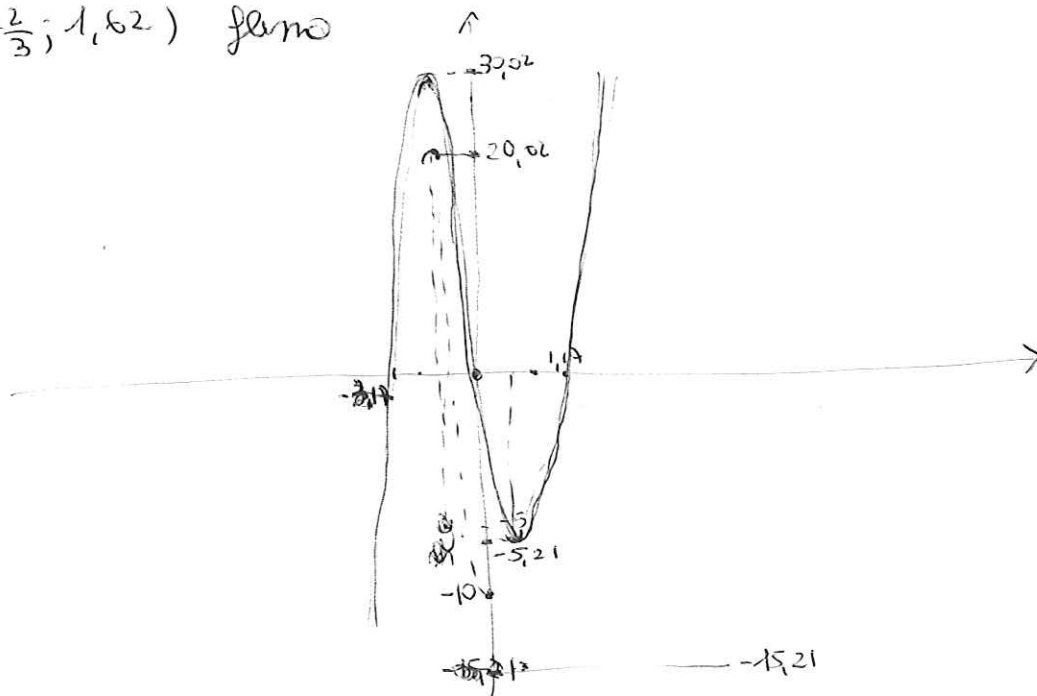


Grafico di $S(x)$ ed $S(x)-10=h(x)$

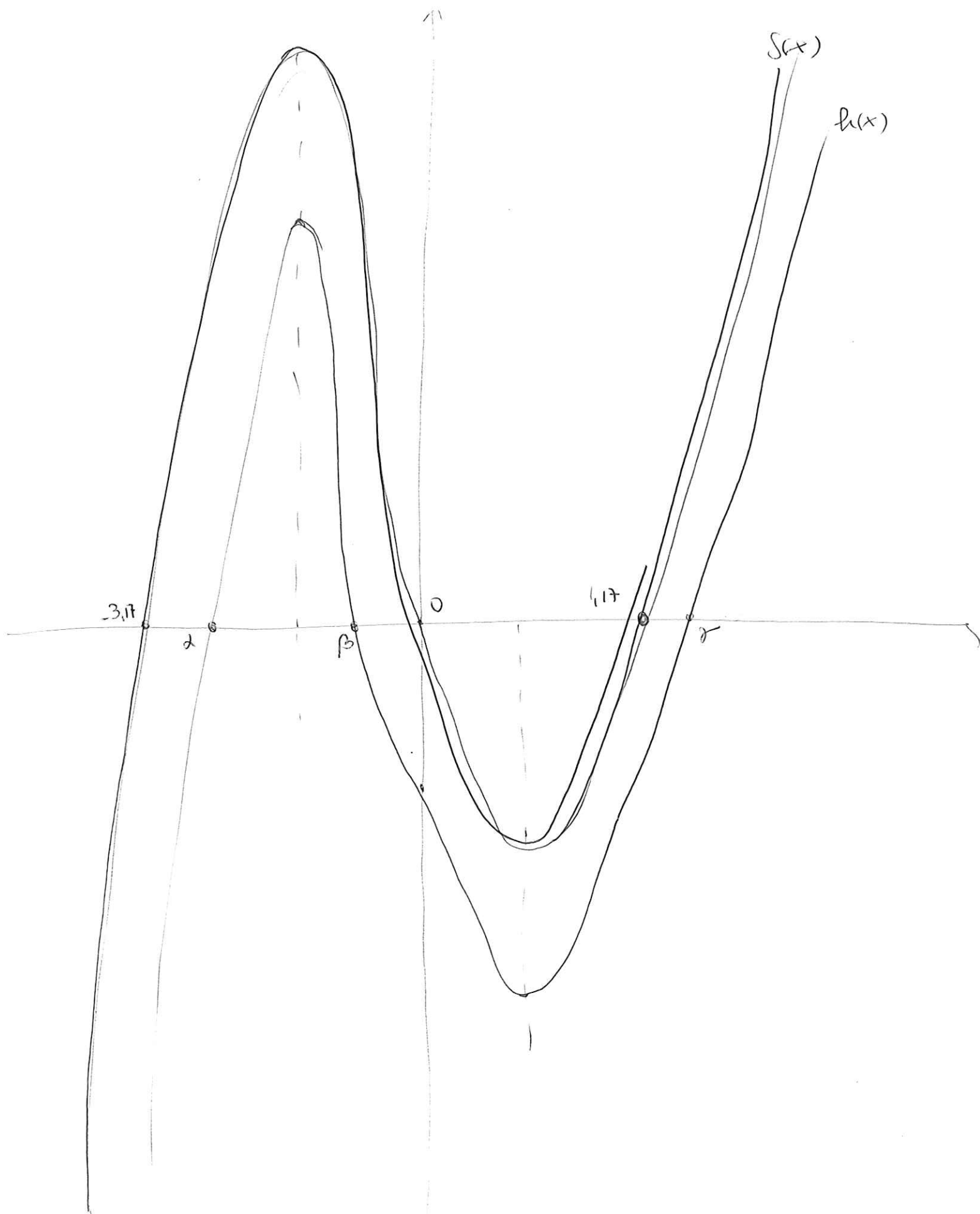
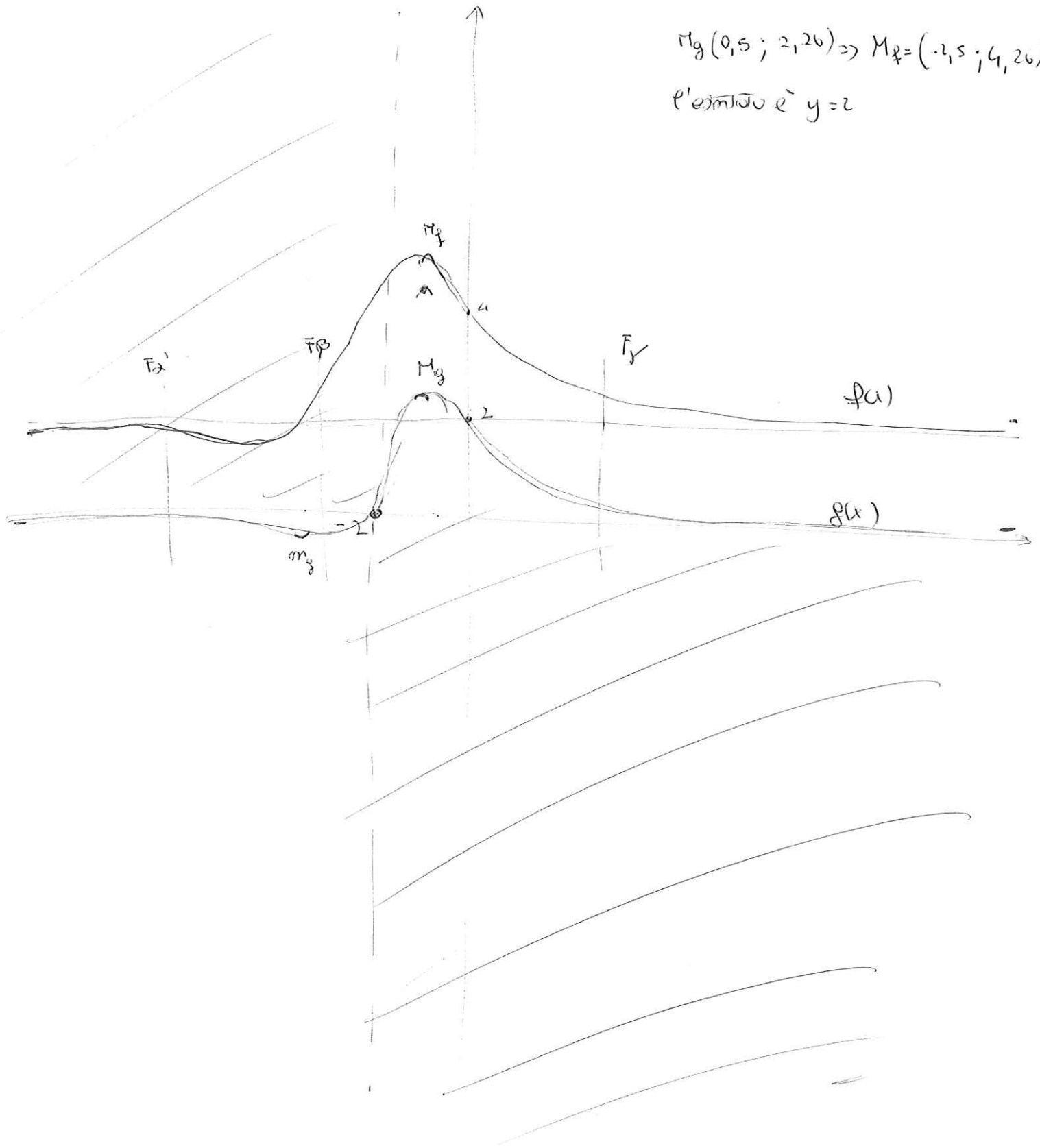


gráfico g(x)

$$m_g(-2,5; -0,04) \Rightarrow m_g = (-2,5; 1,96)$$

$$m_g(0,5; 2,26) \Rightarrow M_g = (-2,5; 4,26)$$

premissa e $y=2$



$\int e^{3x} \sin 3x \, dx =$ per semplificare le ulteriori operazioni
 fare una sostituzione di convenienza
 $3x = t \Rightarrow 3dx = dt \Rightarrow dx = \frac{dt}{3}$

$= \boxed{\frac{1}{3} \int \overbrace{e^t}^{f'(x)} \overbrace{\sin t}^{g(x)} dt} = \quad \left\| \begin{array}{l} f'(x) = e^t \Rightarrow f(x) = e^t \\ g(x) = \sin t \Rightarrow g'(x) = \cos t \end{array} \right\|$

$= \frac{1}{3} \left(e^t \sin t - \int e^t \cos t \, dt \right) = \frac{1}{3} e^t \sin t - \frac{1}{3} \int e^t \cos t \, dt = \quad \left\| \begin{array}{l} f'(x) = e^t \Rightarrow f(x) = e^t \\ g(x) = \cos t \Rightarrow g'(x) = -\sin t \end{array} \right\|$

$= \frac{1}{3} e^t \sin t - \frac{1}{3} \left(e^t \cos t - \int e^t (-\sin t) \, dt \right) =$

$= \boxed{\frac{1}{3} e^t \sin t - \frac{1}{3} e^t \cos t - \frac{1}{3} \int e^t \sin t \, dt} \quad (2)$

quindi ho visto che $(1) = (2)$ cioè:

$\frac{1}{3} \int e^t \sin t \, dt = \frac{1}{3} e^t \sin t - \frac{1}{3} e^t \cos t - \frac{1}{3} \int e^t \sin t \, dt \Rightarrow$

$\frac{1}{3} \int e^t \sin t \, dt + \frac{1}{3} \int e^t \sin t \, dt = \frac{1}{3} e^t \sin t - \frac{1}{3} e^t \cos t \Rightarrow$

$\frac{2}{3} \int e^t \sin t \, dt = \frac{1}{3} e^t \sin t - \frac{1}{3} e^t \cos t \Rightarrow$

$\int e^t \sin t \, dt = \left(\frac{1}{3} e^t \sin t - \frac{1}{3} e^t \cos t \right) \cdot \frac{3}{2} \Rightarrow$

$\Rightarrow \int e^t \sin t \, dt = \frac{1}{2} e^t \sin t - \frac{1}{2} e^t \cos t \Rightarrow$

$\int e^{3x} \sin 3x \cdot 3 \, dx = \frac{1}{2} e^{3x} \sin 3x - \frac{1}{2} e^{3x} \cos 3x$

$3 \int e^{3x} \sin 3x \, dx = \frac{1}{2} e^{3x} \sin 3x - \frac{1}{2} e^{3x} \cos 3x$

$\int e^{3x} \sin 3x \, dx = \frac{1}{6} e^{3x} \sin 3x - \frac{1}{6} e^{3x} \cos 3x + C$

$$\lim_{x \rightarrow 0} \frac{3 - 3\cos^2 x}{2x^2} + (x-1) = \lim_{x \rightarrow 0} \frac{3(1 - \cos^2 x)}{2x^2} + x - 1 =$$

$$= \lim_{x \rightarrow 0} \frac{3}{2} \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} + x - 1 = \frac{3}{2} \lim_{x \rightarrow 0} \underbrace{\frac{\sin x}{x}}_1 \cdot \underbrace{\frac{\sin x}{x}}_1 + x - 1 =$$

$$= \frac{3}{2} \cdot 1 \cdot 1 + 0 - 1 = \frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2}$$

obs. • $\sin^2 x + \cos^2 x = 1 \Rightarrow 1 - \cos^2 x = \sin^2 x$

• $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

DERIVATA

$$f(x) = 3x \cos(\lg x) + 4x \operatorname{sen}(\lg x) - 2x$$

$$\begin{aligned} f'(x) &= (3x)' \cos(\lg x) + 3x (\cos(\lg x))' + (4x)' \operatorname{sen}(\lg x) + 4x (\operatorname{sen}(\lg x))' - (2x)' = \\ &= 3 \cos(\lg x) + 3x (-\operatorname{sen}(\lg x) \cdot \frac{1}{x}) + 4 \operatorname{sen}(\lg x) + 4x \cdot \frac{\cos(\lg x)}{\lg 10 \cdot x} - 2 = \\ &= 3 \cos(\lg x) - 3 \operatorname{sen}(\lg x) + 4 \operatorname{sen}(\lg x) + \frac{4}{\lg 10} \cos(\lg x) - 2 \end{aligned}$$